ERCSC - validation - conversions

Graça Rocha
Caltech
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UNITS

Conversion from MJy/sr to Antenna Temperature (T_AK)

Consider the constants as tabulated in Table I.

TABLE I:

$h =$	$6.626176e-27$ erg * s
$k =$	$1.380662e-16$ erg / K
$c =$	$2.99792458e+10$ cm / s
$T_{cmb} =$	2.726 K

Using definition of Antenna tremperature:

$$T_A = \frac{1}{2k}\left(\frac{c}{\nu}\right)^2 B_\nu \tag{1}$$

where B_ν is the brightness of the source. Using the values of the constants as given in Table I one has:

$$C_{T_AK->MJy/sr} = 2k\left(\left(\frac{\nu}{c}\right)\times 10^9\right)^2 \times 10^{17} = 0.03072387 \times \nu^2 \tag{2}$$

for $\nu = 30$GHz this factor is :

$$C_{T_AK--MJy/sr} = 27.6514857 \tag{3}$$
$$C_{MJy/sr->T_AK} = 3.623 * 10^{-2} \tag{4}$$

this is the factor we've actually used in the conversion for Trieste simulations:
$T_A = 3.623 * 10^{-2} B_\nu(MJy/sr)$

Conversion from Antenna Temperature T_A (RJ) to T- thermodynamic

Use:

- definition of Antenna temperature
- the expression for B_ν, ie the Planck distribution $\hookrightarrow$ differentiate it wrt to temperature :

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{(e^{h\nu/kT}-1)} \tag{5}$$

$$\delta T_A = \frac{c^2}{(2k\nu^2)}\frac{dB_\nu}{dT} = f(\nu)\delta T = \frac{(x^2e^x)}{(e^x-1)^2}\delta T \tag{6}$$

where $x = h\nu/kT$.
Therefore

$$C_{T_A->T_{thermo}} = f = \frac{(e^x - 1)^2}{(x^2 e^x)} \tag{7}$$

$$x = 1.7605556 \times 10^{-11} \times \nu \tag{8}$$

for $\nu = 30GHz$

$$x = 52.816668 \times 10^{-11} \times 10^9 = 0.52816668 \tag{9}$$

$$f = \frac{(e^x - 1)^2}{(x^2 e^x)} = 1.023464 \tag{10}$$

$$C_{T_A->T_{thermo}} = 1.023464 \tag{11}$$

TABLE II: Conversions

μ GHz	μK (thermo) to K (antennaT)	K (antennaT) to MJy/sr	μK (thermo) to MJy/sr	MJy/sr to K(antennaT)
30	9.77074×10^{-7}	27.6515	2.7×10^{-5}	0.0361644
143	6.04833×10^{-7}	628.272	0.0003800	0.00159167
857	6.37740×10^{-11}	22565.1	1.43907×10^{-6}	4.43162×10^{-5}

TABLE III: Conversions

μ GHz	30	143	857
μK_{thermo} to $T_A K$	9.77074×10^{-7}	6.04833×10^{-7}	6.37740×10^{-11}
μK_{thermo} to MJy/sr	2.7×10^{-5}	0.0003800	1.43907×10^{-6}
$T_A K$ to MJy/sr	27.6515	628.271	22565.1
MJy/sr to $T_A K$	0.0361644	0.00159167	4.43162×10^{-5}

TABLE IV: Conversions

μ GHz	30	143	857
K_{thermo} to $T_A K$	0.977074	0.604833	6.37740×10^{-5}
K_{thermo} to MJy/sr	27	380	1.43907
$T_A K$ to MJy/sr	27.6515	628.271	22565.1
MJy/sr to $T_A K$	0.0361644	0.00159167	4.43162×10^{-5}

TABLE V: Conversions

μ GHz	30	44	70	100	143	217	353	545	857
K_{thermo} to T_AK	0.977074	0.951459	0.882496	0.777295	0.604833	0.334417	0.0775445	6.26735×10^{-3}	6.37740×10^{-5}
T_AK to K_{thermo}	1.023468	1.0510271	1.1331761	1.2865727	1.6535001	2.9908492	12.90104	159.67296	15700.02123
K_{thermo} to MJy/sr	27.0175	56.5941	132.857	238.815	380.0	483.82	296.877	57.1943	1.43907
T_AK to MJy/sr	27.6515	59.4814	150.5468	307.2385	628.272	1446.757	3828.469	9125.753	22565.1
MJy/sr to T_AK	0.0361644	0.0168120	0.00664245	0.00325480	0.00159167	0.000691201	0.000261201	0.000109580	4.43162×10^{-5}